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Flight Path Optimization at Constant Altitude

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I. Introduction

MOST analyses of optimal transport aircraft flight begin with the assumption that the flight profile consists of three segments: climb, cruise, and descent. Indeed, this is the flight profile of all long-haul commercial flights today. The dominant stage of such flights, in terms of flight time, is the cruise segment. The air transportation industry is extremely competitive and even small changes in aircraft performance have significant impacts on the operation costs of airlines. Thus there has been, and continues to be, great interest in optimizing the cruising flight of transport aircraft.

The classical performance relation for cruising flight is the "Breguet range equation." This is based on steady flight (constant speed and altitude), leaving only the range and mass as dynamic variables. Integrating the state equations associated with these two variables, assuming a constant lift-to-drag ratio, gives the Breguet equation

$$R = B \ln(m_0/m_f) \tag{1}$$

where R is the range, m_0 is the initial mass, m_f is the final mass, and B is the Breguet factor, given by

$$B = (\lambda V/gC) \tag{2}$$

where λ is the lift-to-drag ratio, V the cruise speed, g the gravitational acceleration, and C the thrust specific fuel consumption.

Thus, to optimize the flight path (in the sense of either maximizing range for a given mass ratio, or maximizing the mass ratio for a given range), a search is conducted to find the point in the flight envelope (the portion of the (h, V) plane that does not violate any constraints) that maximizes B. Because the Breguet factor changes as fuel is burned off during the flight, the optimal (h, V) values change as well. Typically the optimum altitude increases during the flight, resulting in a steady "cruise climb." Air traffic control requires that aircraft hold specific altitudes; thus the operational flight paths of long-haul transport aircraft are "step climbs" during which the altitude is increased at discrete times. Note that a cruise or step climb violates the assumption of flight at constant h, but the rate of change of altitude is quite small.

Several authors have used more detailed math models to study aircraft cruise, models in which V and h are allowed to vary [1,2]. These authors have investigated whether or not cyclic cruise is better than steady cruise. It was found that flight paths with large periodic changes in h, V, and throttle could be more fuel efficient. It was found, however, that the improvement (reduction) in fuel consumption was very small, 1% at best. Furthermore, such flight paths would not be compatible with air traffic procedures as the altitude oscillations sometimes exceed 10,000 ft.

The work just discussed focuses on the interplay between h, V, and throttle setting. Because of the operational restrictions on cruise altitude, in the present paper we look at aircraft cruise from a different point of view; in particular, we study the interplay between aircraft mass and speed at constant altitude. Because this is a singular optimal control problem, we begin with a review of that subject.

II. Singular Optimal Control

Consider a system whose math model is a set of state equations

$$\dot{x} = f(x) + g(x)u \tag{3}$$

where $x \in \Re^n$ is the state vector and $u \in \Re$ is the scalar control variable bounded by $u_m \le u \le u_M$. The state may be free or fixed at $t = 0$ and $t = t_f$. It is desired to minimize

$$J = \int_0^{t_f} [f_0(x) + g_0(x)u]\,\mathrm{d}t \tag{4}$$

This is a problem of singular optimal control [3,4]. This theory will now be reviewed.

The Hamiltonian of the system is

$$H = \lambda_0 f_0 + \lambda_0 g_0 u + \lambda f + \lambda g u \tag{5}$$

Next, relabel

$$\lambda = \begin{pmatrix} \lambda_0 \\ \lambda_1 \\ \vdots \\ \lambda_n \end{pmatrix}, \qquad f = \begin{pmatrix} f_0 \\ f_1 \\ \vdots \\ f_n \end{pmatrix}, \qquad g = \begin{pmatrix} g_0 \\ g_1 \\ \vdots \\ g_n \end{pmatrix} \tag{6}$$

With this notation, and setting $\lambda_0 = 1$ (one of the necessary conditions of optimal control, assuming that $\lambda_0 \ne 0$), Equation (5) becomes

$$H = \lambda f + \lambda g u \tag{7}$$

The adjoint variables λ are defined by

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$$\dot{\lambda} = -H_x = -\lambda f_x - \lambda g_x u \tag{8}$$

where subscripts denote derivatives.

Now write H as the sum of two terms, one part depending on the control and one not;

$$H = \bar{H} + Su \tag{9}$$

where

$$\bar{H} = \lambda f, \qquad S = \lambda g \tag{10}$$

Two of the necessary conditions for optimal control (minimum principle) are that the optimal control minimizes H and that it gives $H = 0$. The first of these gives the optimal control law as

$$u = \left\{ \begin{array}{ll} u_M, & \text{if } S < 0 \\ u_{\text{sing}}, & \text{if } S = 0 \\ u_m, & \text{if } S > 0 \end{array} \right\} \tag{11}$$

The function S is called the switching function, for obvious reasons.

In Eq. (11) the possibility that S is identically zero for some nonzero interval of time has been allowed for. This is the case of singular control. In this case the minimum principle does not provide the optimal control, but this may be determined as follows. We have $H = 0$ and, by hypothesis, $S = 0$ on an interval of time; on this interval, $\dot{S} = 0$ as well. The following three equations are usually sufficient to determine the singular arc in terms of the state variables:

$$\bar{H} = \lambda f = 0, \qquad S = \lambda g = 0, \qquad \dot{S} = -\lambda f_x g + \lambda g_x f = 0 \tag{12}$$

There is an additional necessary condition, sometimes called the Kelley condition, which optimal singular control must satisfy. First, note that Eqs. (12) do not contain the control explicitly. It turns out that the control can appear only on even derivatives of S. Let

$$\overset{(i)}{S} = \frac{d^i S}{dt^i} = 0 \text{ is not a function of } u; \quad i = 0, 1, \cdots, m-1$$
$$\overset{(m)}{S} = a(x, \lambda) + b(x, \lambda)u = 0 \tag{13}$$

with m even. The Kelley condition [4,5] says that for a singular control to be optimal, we must have

$$(-1)^{\frac{m}{2}}(\partial/\partial u)\overset{(m)}{S} = (-1)^{\frac{m}{2}} b(x, \lambda) \geq 0 \tag{14}$$

Equation (13) also gives an explicit equation for the singular control, when the adjoint variables have been eliminated, provided $u_m \leq u_{\text{sing}} \leq u_M$:

$$u_{\text{sing}} = -\frac{a(x, \lambda)}{b(x, \lambda)} = -\frac{a'(x)}{b'(x)} \tag{15}$$

There is an alternative method of finding the singular arc that employs Green's theorem, due to Mancill [6] and developed by Miele [7]. This applies only to the case of two state variables, say x and y, the case considered later in this Note. Now consider the optimization problem with x and y fixed at $t = 0$ and $t = t_f$

$$\dot{x} = f_x(x, y) + g_y(x, y)u, \qquad \dot{y} = f_y(x, y) + g_y(x, y)u$$
$$J = \int_0^{t_f} f_0(x, y)\,\mathrm{d}t \tag{16}$$

Eliminating the control between the state equations and substituting into the cost functional results in

$$J = \int_{(x_0, y_0)}^{(x_f, y_f)} (A\,\mathrm{d}x + B\,\mathrm{d}y) \tag{17}$$

where

$$A = \frac{f_0 g_y}{f_x g_y - f_y g_x}, \qquad B = \frac{f_0 g_x}{f_y g_x - f_x g_y} \tag{18}$$

Equation (17) is a line integral in the plane. Green's theorem relates line integrals around closed curves to area integrals. To use the theorem, consider the closed curve consisting of the curve to be optimized, C_1, plus a fixed, but arbitrary, curve, C_2, returning to the starting point; then Green's theorem is

$$\int_{C_1} (A\,\mathrm{d}x + B\,\mathrm{d}y) + \int_{C_2} (A\,\mathrm{d}x + B\,\mathrm{d}y) = \iint_A \left(\frac{\partial A}{\partial y} - \frac{\partial B}{\partial x}\right) \mathrm{d}A \tag{19}$$

Because integral C_2 is fixed, minimizing integral C_1 is the same as minimizing the integral A. The critical curve associated with the latter integral is

$$(\partial A/\partial y) - (\partial B/\partial x) = 0 \tag{20}$$

and is minimizing. This equation is the singular arc and is equivalent to solving Eqs. (12) simultaneously and eliminating the adjoints.

III. Cruise Problem

Consider an aircraft flying at constant altitude at a constant heading. The equations of motion are

$$\dot{V} = [(T - D)/m], \qquad \dot{m} = -\beta, \qquad L = mg \tag{21}$$

where V is the speed; m is the mass; T, D, and L are the thrust, drag, and lift forces, respectively, and β is the fuel flow rate. It is assumed that

$$T(V) = \Pi T_M(V), \qquad \beta(V) = C(V)T(V)$$
$$D = AV^2 + (BL^2/V^2) \tag{22}$$

where D represents a parabolic drag polar [8], $A = C_{D0}(\rho s/2)$, and $B = (2K/\rho s)$. The zero-lift drag coefficient C_{D0}, the induced drag coefficient K, the air density ρ, and the reference area s are all taken as positive constants, a good assumption for flight at constant altitude of a subsonic aircraft. Π is the throttle setting where $\Pi_m \leq \Pi \leq 1$. It is desired to maximize the range

$$J = -\int_0^{t_f} V\,\mathrm{d}t \tag{23}$$

with t_f free. This is a problem with two states, V and m, and one control, Π. This is very similar to the problem presented in Sec. 14.10 of Leitmann [3], and considered earlier by Miele [9]. The difference is that instead of a rocket engine with constant exhaust exit velocity, we have an air breathing engine with speed-dependant maximum thrust T_M and specific fuel consumption C.

The Hamiltonian and adjoint equations are

$$H = -V + \lambda_V\left(\frac{\Pi T_M}{m} - \frac{AV^2}{m} - \frac{Bmg^2}{V^2}\right) - \lambda_m C \Pi T_M$$
$$\dot{\lambda}_V = 1 + \lambda_V\left(-\frac{\Pi T_{M_V}}{m} + \frac{2AV}{m} - \frac{2Bmg^2}{V^3}\right) + \lambda_m(C_V \Pi T_M + C \Pi T_{M_V})$$
$$\dot{\lambda}_m = \lambda_V\left(\frac{\Pi T_M}{m^2} - \frac{AV^2}{m^2} + \frac{Bg^2}{V^2}\right) \tag{24}$$

where $C_V = (dC/dV)$ and $T_{M_V} = (dT_M/dV)$.

On the singular arc,

$$\bar{H} = -V + \lambda_V\left(-\frac{AV^2}{m} - \frac{Bmg^2}{V^2}\right) = 0, \qquad S = \frac{\lambda_V}{m} - \lambda_m C = 0$$

$$\dot{S} = 1 + \lambda_V\left(\frac{2AV}{m} - \frac{2Bmg^2}{V^3} + \frac{CAV^2}{m} - \frac{CBmg^2}{V^2}\right) + \lambda_m C_V\left(AV^2 + \frac{Bm^2g^2}{V^2}\right) = 0 \tag{25}$$

Note that the T_M and T_{M_V} terms have canceled out. Eliminating the adjoints from Eqs. (25) gives the singular arc

$$m = \frac{V^2}{g}\sqrt{\frac{A[1 + VC + (V/C)C_V]}{B[3 + VC - (V/C)C_V]}} \tag{26}$$

Note that if C is a constant, this equation reduces to that derived by Leitmann [3].

To obtain this result using Green's theorem, we begin by eliminating the control from Eqs. (21) and substituting the result into Eq. (23); the result is

$$J = \int_{(V_0,m_0)}^{(V_f,m_f)}\left(\frac{mV}{D}\,\mathrm{d}V + \frac{V}{CD}\,\mathrm{d}m\right) \tag{27}$$

Applying Green's theorem, this is equivalent to

$$J = \iint_A \left[\frac{\partial}{\partial m}\left(\frac{mV}{D}\right) - \frac{\partial}{\partial V}\left(\frac{V}{CD}\right)\right]\mathrm{d}V\,\mathrm{d}m \tag{28}$$

Thus the singular arc is

$$[(\partial/\partial m)(mV/D) - (\partial/\partial V)(V/CD)] = 0 \tag{29}$$

Carrying out the differentiation gives Eq. (26); this method is easier than using Eqs. (25).

To check the Kelley necessary condition, Eq. (14), and to derive an expression for the thrust needed to follow the singular arc, Eq. (15), $\ddot{S} = 0$ must be formed. This expression turns out to be algebraically intractable (41 terms); however, it can be established that the thrust occurs in this equation so that $m = 2$ in Eqs. (13) and (14). Thus Eq. (14) becomes

$$b(x, \lambda) \le 0 \tag{30}$$

There are two special cases where $\ddot{S} = 0$ can be solved to give results. First, suppose that the drag due to lift is independent of V (that is $D = AV^2 + B_1L^2$) and C is independent of V as well. Then we have

$$m = \frac{V}{g}\sqrt{\frac{A}{B_1}}, \qquad T = \frac{2AV^2}{1 + CV} = \frac{D}{1 + CV}, \qquad b = -\frac{B_1g^2}{AV^3}(1 + CV)^2 \tag{31}$$

so that the Kelley necessary condition, Eq. (30), is strictly satisfied.

The second case, of more practical interest, comes from assuming that the term CV is small compared with 1 (a very good assumption for subsonic transport aircraft as will be discussed in the next section) and C is independent of V; the drag remains given by Eq. (22). The results are

$$m = \frac{V^2}{g}\sqrt{\frac{A}{3B}}, \qquad T = \tfrac{4}{3}AV^2, \qquad b = -\frac{9Bg^2}{2AV^5} \tag{32}$$

so that again Eq. (30) is satisfied. It is interesting to note that for this case, thrust equals drag and the zero-lift drag is 3 times the drag due to lift. (The classical result is zero-lift drag equal to the drag due to lift.)

IV. Specific Fuel Consumption

Aircraft data show that specific fuel consumption is far from constant and can vary significantly over an aircraft's flight envelope. The equations for C are very complicated and highly specific to the engine. C is mainly a function of speed and air temperature. Altitude is a factor in a C calculation, only as air temperature varies with altitude.

For turbojets and turbofans, C depends on several temperatures in the engine, pressure ratios, bypass ratios, and the fuel to air mass ratio in the combustor. There are computer simulation capabilities that can provide the information we want, and here we use the NASA Glenn EngineSim.‡

The NASA Glenn EngineSim, setup for a CF6 turbofan sized for use on a 747-400, was sampled at various velocities to generate a C vs V relationship for full throttle of the engine. Another condition was generated, where the engine thrust matches the drag of the aircraft with respect to speed, which is more accurate to what an engine mounted on an aircraft would provide. This was done by developing a table of speed and parabolic drag values, entering the speed into EngineSim, and then adjusting the throttle to match the drag.

Figure 1 shows these results for the general relation of C as a function of speed at constant altitude. The dashed line represents the full throttle condition, and as expected, the slower the aircraft is flying, the more efficient the turbofan becomes. The solid line represents the change to specific fuel consumption if, in addition to the reduced speed, the throttle is reduced to match the drag. Notice that minimum throttle occurs at about 250 m/s.

It is possible to simplify Eq. (26) by considering values of the VC term. Subsonic aircraft speed does not get much above 300 m/s; however, C can vary significantly depending upon the type of engine and application. As will be discussed later in this paper, the CF6-80C2 engine example would have a C around 0.04 kg/N · h, which would be around 10^{-5} s/m. Therefore VC, a dimensionless number, would be on the order of 3×10^{-3}, which is small compared with 1 and 3. To give an order of scale to the VC value, the Concorde flying at Mach 2 has a VC value of only 0.018. The VC for an F-16 at full afterburner is around 0.03. For the VC term to be really large, the aircraft would need to have high speed and be using rocket engines. The space shuttle main engines have a VC of 1.67. [The space shuttle VC value is approximated by orbital speed divided by specific impulse and gravity. Specific impulse for the space shuttle main engines (SSME) is 428 s.] Thus the VC term may be neglected for our application.

V. Example

For example calculations, a Boeing 747-400 with General Electric CF6-80C2 high bypass, multistage, turbofan engines is selected. Depending upon the options installed on the engine, the CF6-80C2 is rated to produce maximum thrust of 282,500 N per engine (63,500 lbf).§

The design range of a 747-400 is listed as 13,444 km (8,354 miles), with a maximum takeoff mass of 362,875 kg (800,000 lb)¶ [10]. At takeoff, a full fuel load would be 120,205 kg (265,000 lb). This leaves 61,415 kg (135,400 lb) for passengers, crew and cargo. It is easy to see that even small percentage changes to the amount of fuel required for each flight can provide significant potential for increased payload and thus for cost savings and increased revenue.

For the 747-400 aircraft and CF6-80C2 engine model, we input speed and throttle values to get C, thereby getting an order of

‡NASA Glenn EngineSim, Version 1.6e (online application); http://www.grc.nasa.gov/WWW/K-12/airplane/ngnsim.html [retrieved 2 December 2004].

§General Electric Aircraft Engines website, GE Transportation–Aircraft Engines: CF6. http://www.geae.com/engines/commercial/cf6/cf6-80c2.html [retrieved 2 December 2004].

¶Boeing Aircraft Company website, Boeing 747 Family, http://www.boeing.com/commercial/747family/technology.html [retrieved 2 December 2004].

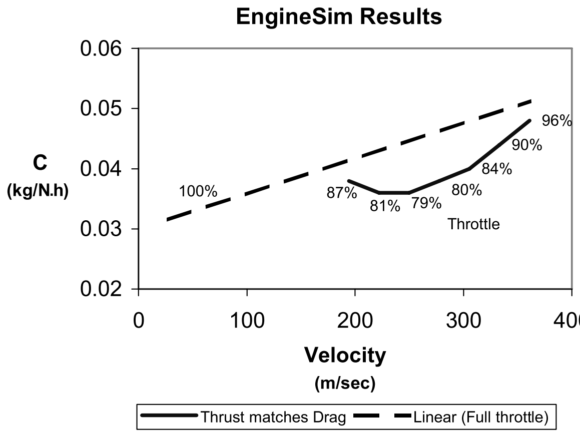

Fig. 1 C variation with respect to speed for a high bypass turbofan.

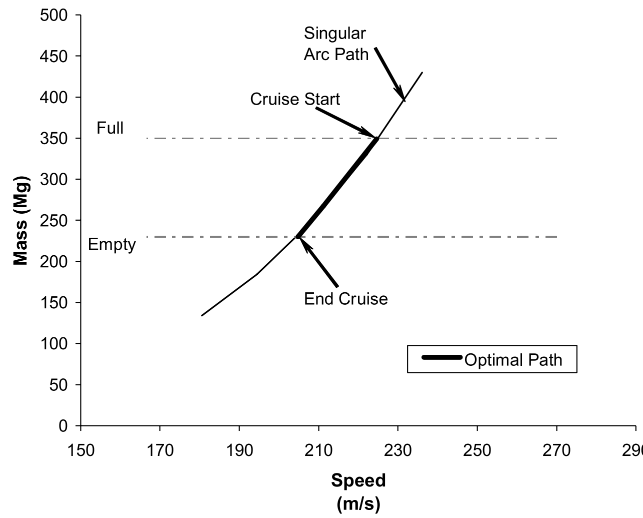

Fig. 2 Singular arc and optimal path for 747-400.

magnitude of C_V to determine if it is significant or not. By using two mass-speed points, entering them in to EngineSim, adjusting throttle, and looking at the change of C, we find C_V is on the order of 10^{-8}. With V/C being on the order of 10^8, the term $(V/C)C_V$ is significant. Therefore, after eliminating VC, but leaving $(V/C)C_V$, Eq. (26) becomes

$$m = \frac{V^2}{g}\sqrt{\frac{A[1 + (V/C)C_V]}{B[3 - (V/C)C_V]}} \tag{33}$$

It is interesting to note that if C is linear with respect to V, the term $(V/C)C_V$ becomes equal to 1, which would then equal the classical solution. Using EngineSim, this is found not to be the case for the 747-400 type aircraft modeled, which has a $(V/C)C_V$ term that varies from 1.21 to 0.93.

We now conjecture how optimal paths look in the (m, V) plane, with Eq. (33) for the 747-400 plotted on Fig. 2. The aircraft climbs to the start of the cruise on the singular arc, follows the arc until cruise fuel is expended and then descends. For our example, the entire cruise portion of the flight may follow the singular arc. A 747-400 with maximum fuel load flying the singular arc path for the entire cruise portion of the flight, starts out burning about 10.0 kg/km, and finishes burning 6.53 kg/km. In contrast, the constant speed path at typical cruise, 250 m/s, would start out burning 10.4 kg/km and finish burning 7.62 kg/km.

A computer simulation of the flight's cruise portion shows that following the singular arc would result in reduced fuel consumption and additional potential range. The steady cruise is performed at 250 m/s and both the singular arc and steady cruise are at 10,000 m altitude. At extreme range, the optimal path could save 8,800 kg (19,360 lb) of fuel, a 7.3% fuel savings. Alternatively, the range of the 747-400 could be increased by 1,325 km (820 miles), an additional 9.8% over the constant speed range, but consuming the same fuel load. Table 1 provides comparative details resulting from the flight modeling. Longer flight time is the main drawback of the optimal path. The cruise portion of a minimum fuel flight following the singular arc would require about 17.5 h versus 15 h for the constant speed cruise. This may be of concern for passenger airline operations.

Time history plots (Fig. 3) that compare the optimal path with that for standard cruise also show the increased range and/or fuel conservation of the optimal path, as well as comparing the flight speed and thrust. One interesting finding is that the optimal path results in higher output thrust during most of the flight time. (See thrust, Fig. 3.) This would tend to disagree with the fuel savings finding, except that the aircraft is flying at a more efficient speed for the large turbofan engines. In this manner, the engines require less fuel to produce each unit of thrust, and can therefore consume less fuel per unit distance flown even though the thrust is greater than for the standard cruise.

VI. Conclusions and Discussion

We have analyzed aircraft cruise at constant altitude with a model with mass and speed as state variables. This is a singular optimal control problem and we have identified the singular arc. Example calculations using the Boeing 747-400 with General Electric CF6-80C2 engines show that the fuel savings of following the singular arc are about 7% relative to the current constant cruise speed. The entire singular arc is in the flight envelop and therefore can be flown.

Further study is required to determine the actual fuel savings over an entire flight. First, the boundary conditions on the cruise portion of the flight are different from the steady cruise case. For example, the singular-arc cruise ends at a lower speed than it begins and thus the range gained in descent will be shorter. Thus the fuel consumption in climb and descent must be added to give a fair comparison. Second, the steady cruise flight path would consume less fuel at slower speeds than the current cruise speed, which was used in the comparison. Additional comparison should be made with a steady cruise speed selected for minimal fuel consumption.

There are obvious air traffic control issues with flying singular-arc flight paths in controlled airspace. Mixing singular-arc paths with steady cruise paths is clearly not acceptable. Even with all aircraft

Table 1 Comparative results for optimal path and standard cruise

Flight profile	Cruise speed, m/s	Range flown, km	Fuel used, kg	Flight time, h
Fixed range				
Standard cruise	250	13,570	120,200	15.1
Optimal path	222–210	13,570	111,400	17.5
Maximum range				
Standard cruise	250	13,570	120,200	15.1
Optimal path	222–209	14,895	120,200	19.25

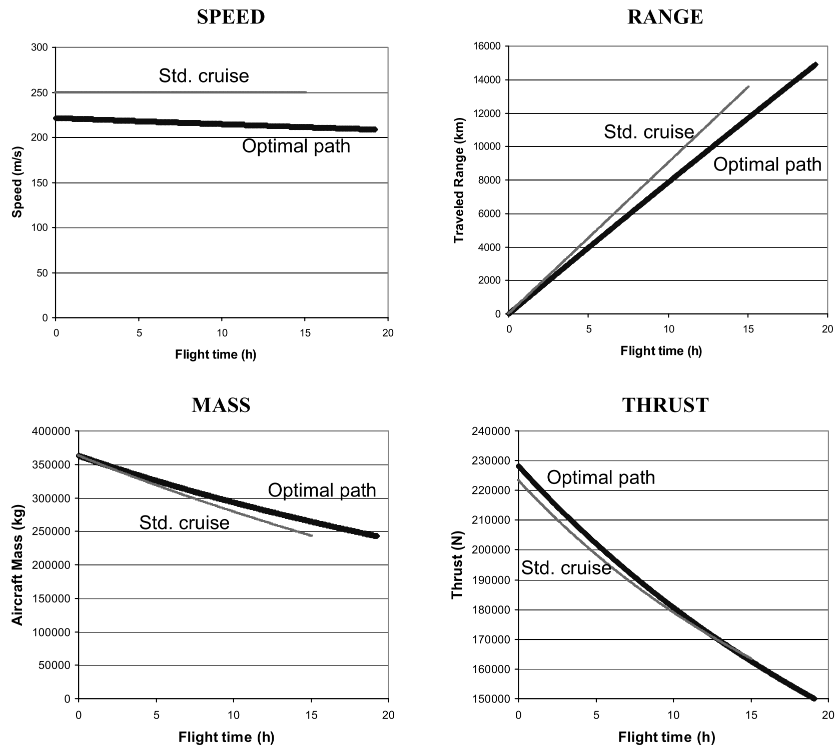

Fig. 3 Comparative time history plots of optimal path and standard cruise.

flying singular-arc paths there is the issue of separating aircraft that are decelerating. In many parts of the world, however, airspace is not controlled and flight time is not critical. In such situations, singular-arc flight may be employed immediately.

Perhaps the most important application of our results is to aircraft design. Key aircraft parameters (such as wing loading, aspect ration, and wing sweep) could be chosen to move the singular arc to its optimum location in the mass-speed plane. The performance criteria should be a suitably weighted combination of fuel consumption and flight time so as to minimize direct operating cost.

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